What is Scarlet Fever?

The Organism:
Scarlet fever is caused by the group A streptococcal bacterium, streptococcus pyogenes. Group A streptococcal bacteria cause a wide range of infections from strep throat to necrotizing fasciitis (flesh-eating disease) and streptococcal toxic shock syndrome (STSS).

The Infection:
Two to three days after the direct passage of droplets containing scarlet fever producing streptococcus pyogenes bacteria, the patient develops a sore throat and fever. A rash characterized by dots the size of pinheads that give the skin a sandpaper-like texture also develops on the first day of the fever. There is general redness of the face and forehead and on the tongue, inflamed bumps rise above a white coating that then changes to bright red (called "strawberry tongue"). Without treatment, the infection usually lasts about 6 days; however, antibiotics can rapidly suppress the growth of group A streptococcal bacteria within 24 hours. Without antibiotic therapy, rheumatic fever can develop about 18 days after the scarlet fever infection. Rheumatic fever is a dangerous complication and can damage the heart valves.

Prevalence:
Scarlet fever was at one time a common childhood illness. In the 19th century it was a dreaded and sometimes fatal disease. However by the 1920s its severity and frequency had waned. Today scarlet fever is a rare infection.

The Basic SEIR Epidemic Model

The basic SEIR epidemic model divides the population up into four compartments: those people that are susceptible to the given disease, those that have been Exposed to the disease but are not yet contagious, those that are Infected and contagious, and those that are Recovered and immune. The total population is N = S + E + I + R and the movement of individuals between the four compartments can be described by four differential equations:

\[
\frac{dS}{dt} = -βSI + \mu (N - S) \quad \frac{dE}{dt} = βSI - (γ + μ)I
\]

\[
\frac{dI}{dt} = (γ + μ)I - (γ + μ)E - (γ + μ)I \quad \frac{dR}{dt} = (γ + μ)E - (γ + μ)R
\]

where β is the transmission rate, 1/γ is the latency period, 1/μ is the infective period, and μ is both birth rate and death rate.

Appropriate values for μ and γ for scarlet fever were estimated from the literature. The value for μ was estimated using data listing the number of births per month during the 1924-1955 time period. Three values for γ were estimated using a method based on age structured case data and these γ values span the complete range of realistic values for μ. Plots for the three values of γ are shown in figure 4.

A list of the estimated parameters is given below:

- Birth rate (μ): 0.026 year⁻¹
- Latent period (1/γ): 3 days
- Infectious period (1/μ): 6 days
- Transmission rate (β): 151.296 2016 year⁻¹

Figure 4: The basic SEIR model plotted for 150 years for the three estimates of β. The oscillations seen in μ have a period of 4.8 years, in (γ) 3.7 years, and in (μ) 2.7 years.

The periods observed in the basic SEIR models are not consistent with the periods observed in the data.

The Seasonally Forced SEIR Epidemic Model

The seasonally forced SEIR model is like the basic SEIR model, but the constant γ (transmission rate) is replaced by a periodic function of time, (μ(t)). In this study we use a function for (μ(t)) based on the school calendar, by taking μ(t) to be a constant high value on school days and a constant low value on non-school days. Other than the parameters used were those calculated for the basic SEIR model. Two different estimates of the amplitude of the function (μ(t)) were made following the method described by Bauch and Earn, 2003 and figure 5 shows the output of the model at those estimates.

Figure 5: The seasonally forced SEIR model plotted for 150 years (a and c) and for the last ten years of that 150 year time period (b and d).

The annual period observed in the seasonally forced SEIR models is consistent with the annual period observed in the data.

Conclusions

There are two main observed oscillatory patterns in the number of scarlet fever cases in Canada from 1924 to 1955; a period of 1 year and a period of 6.25 years. The basic SEIR model does not explain either of these patterns; however, the seasonally forced SEIR epidemic model is consistent with the observed period of 1 year (see summary table below). The observed period of 6.25 years does not appear to be explained by either of the models examined, but it is encouraging that the 6.25 year period is dominant in the less densely populated west provinces. This suggests that further analysis of the seasonally forced SEIR model using methods discussed by Bauch and Earn, 2003 may show that the 6.25 year period can be explained by the seasonally forced SEIR model when a sufficient level of demographic stochasticity is considered.

References